KSU CET

S1 & S2 Notes

2019 Scheme

Manifest the leadership in you

13/8/2019

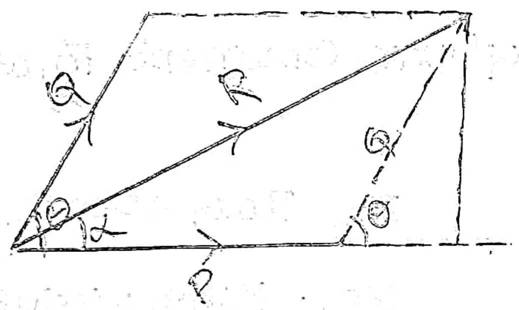
ENGINEERING MECHANICS

MODULE-1

RESULTANT OF FORCES

1. Parallelogram Law of Forces

If two forces acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.

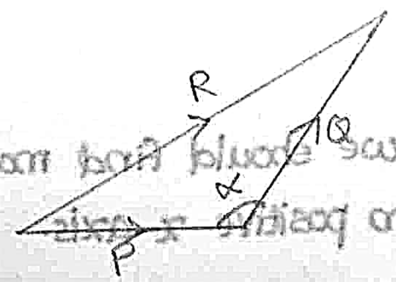


$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

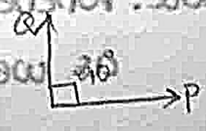
2. Triangle Law of Forces

If the two forces acting on a body are represented in magnitude and direction as two sides of a Δ^{le} in order then the third side or the closing side of the Δ^{le} would be the resultant in opposite order.



$$R = \sqrt{P^2 + Q^2 - 2PQ\sin\theta}$$

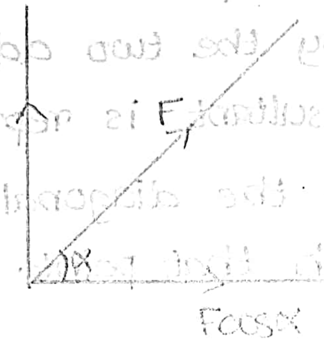
$R = \sqrt{P^2 + Q^2}$, if $\alpha = 90^\circ$



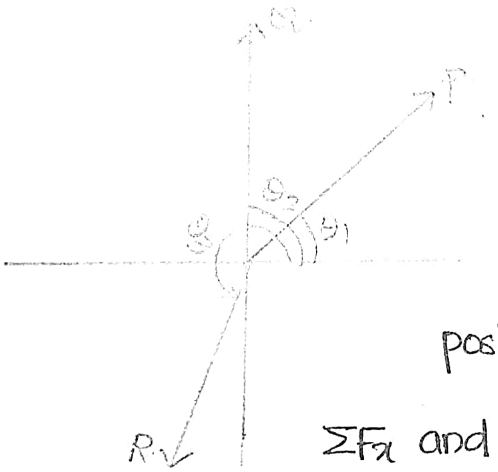
Resultant:

A single force that will replace a system of forces and produces the same effect on the rigid body as that of the system of forces.

Resolution of Forces



To Find the Resultant of Coplanar Concurrent Forces

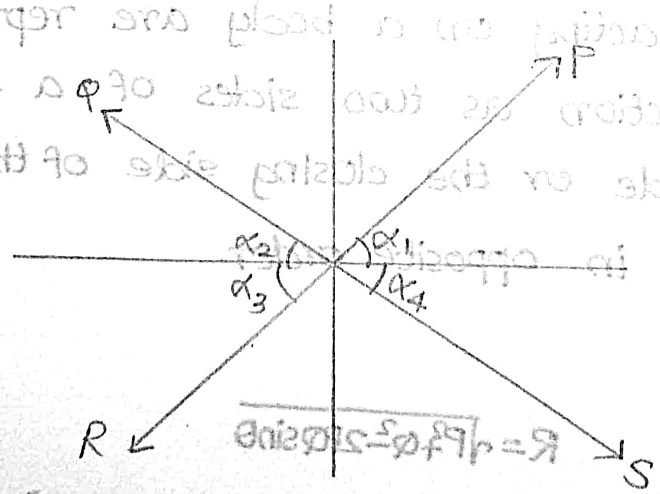


$$\Sigma F_x = P\cos\theta_1 + Q\cos\theta_2 + R\cos\theta_3 \ldots$$

$$\Sigma F_y = P\sin\theta_1 + Q\sin\theta_2 + R\sin\theta_3 \ldots$$

where, $\theta_1, \theta_2, \theta_3$ are angles from positive x-axis.

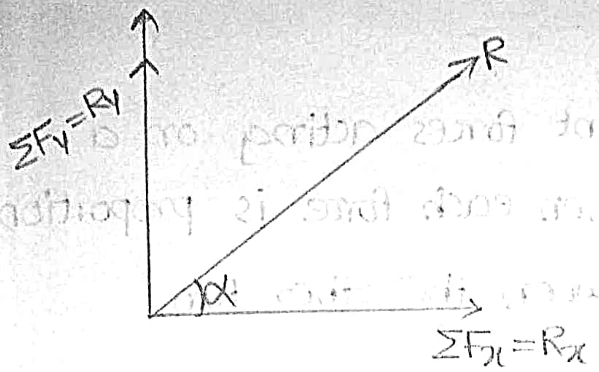
ΣF_x and ΣF_y are components of resultant along x and y-direction.



$$\Sigma F_x = P\cos\alpha_1 - Q\cos\alpha_2 - R\cos\alpha_3 + S\cos\alpha_4$$

$$\Sigma F_y = P\sin\alpha_1 + Q\sin\alpha_2 - R\sin\alpha_3 - S\sin\alpha_4$$

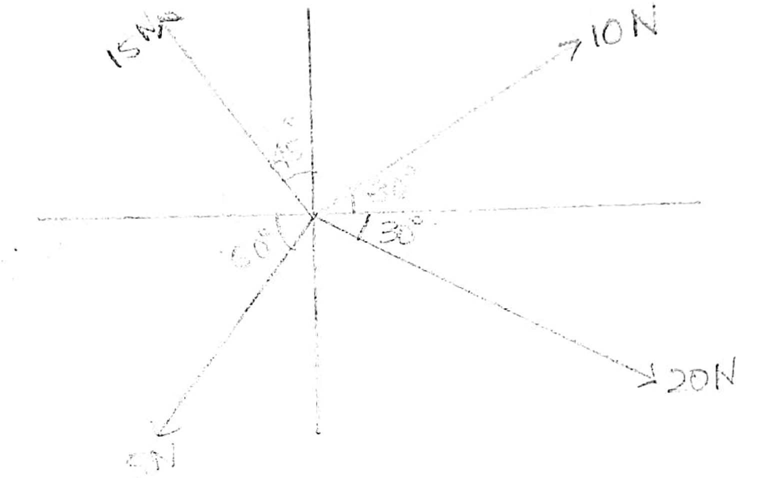
Note: Whenever we say resultant, we should find magnitude as well as direction taken from positive x-axis.



$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x}$$

Q: Find the resultant of given system of forces.



$$\Sigma F_x = 10\cos 30^\circ - 15\sin 45^\circ - 5\cos 60^\circ + 20\cos 30^\circ$$

$$\cancel{= 12.049\ N} \quad = 12.874\ N$$

$$\Sigma F_y = 10\sin 30^\circ + 15\cos 45^\circ - 20\sin 30^\circ - 5\sin 60^\circ$$

$$= 1.8276\ N$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \cancel{14.329\ N} \quad 12.937\ N$$

$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x} = \frac{1.8276}{12.874} = 0.099$$

$$\cancel{\alpha = \tan^{-1} 0.2007 = 12.609^\circ} \quad \alpha = \tan^{-1}(0.099) = 5.653^\circ$$

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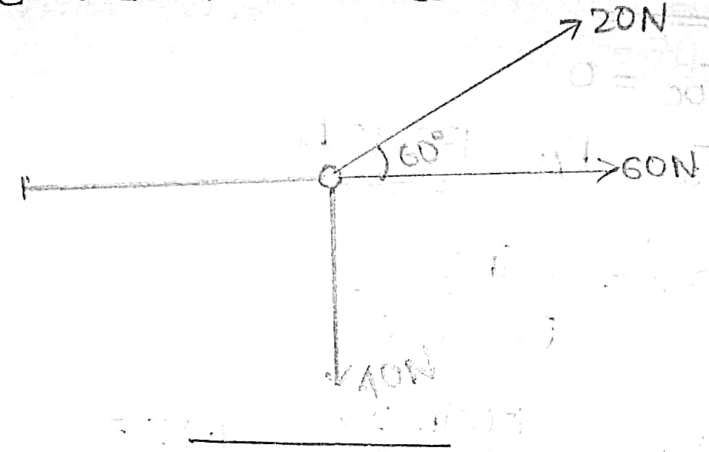
EQUILIBRIUM OF RIGID BODIES

A rigid body is said to be in equilibrium if the resultant of all external and reactive forces and moments acting on it is zero.

Q: Three wires exert tension on a string as shown in figure. Determine the force on a single wire which will replace the three wires.



$$\Sigma F_x = 60 + 20\cos 60^\circ = 60 + 20 \times \frac{1}{2} = \underline{\underline{70N}}$$

$$\Sigma F_y = 20\sin 60^\circ - 40 = 10\sqrt{3} - 40 = \underline{\underline{-22.68\,N}}$$

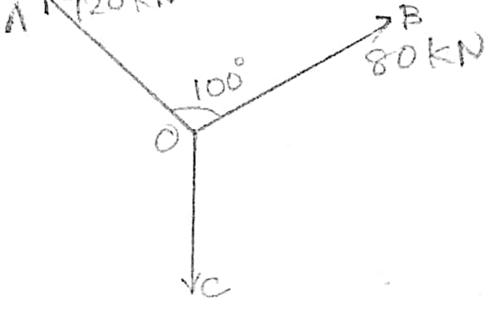
$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{(70)^2 + (22.68)^2} = \underline{\underline{73.582\,N}}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \underline{\underline{17.952^\circ}}$$

Condition of Equilibrium of Coplanar Concurrent Forces

$$R = 0$$
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

Q: Three forces OA, OB and OC are acting at a point such that ∠AOB is 100°. Force OA and OB are 120 kN and 80 kN respectively. Find the magnitude and direction of force OC to keep the point O in equilibrium.



$$R_{AB} = \sqrt{(120)^2 + 80^2 + 2 \times 120 \times 80 \cos 100^\circ}$$

$$= 132.15\ N$$

$$R_{AB} - F_{OC} = 0$$

$$F_{OC} = R_{AB} = 132.15\ N$$

$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\tan \alpha = \frac{120 \sin 100^\circ}{80 + 120 \cos 100^\circ} = 1.9975$$

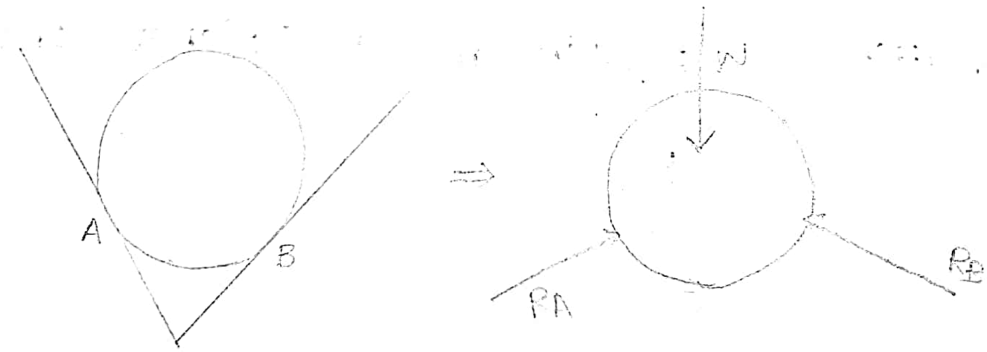
$$\alpha = \tan^{-1}(1.9975) = 63.406^\circ$$

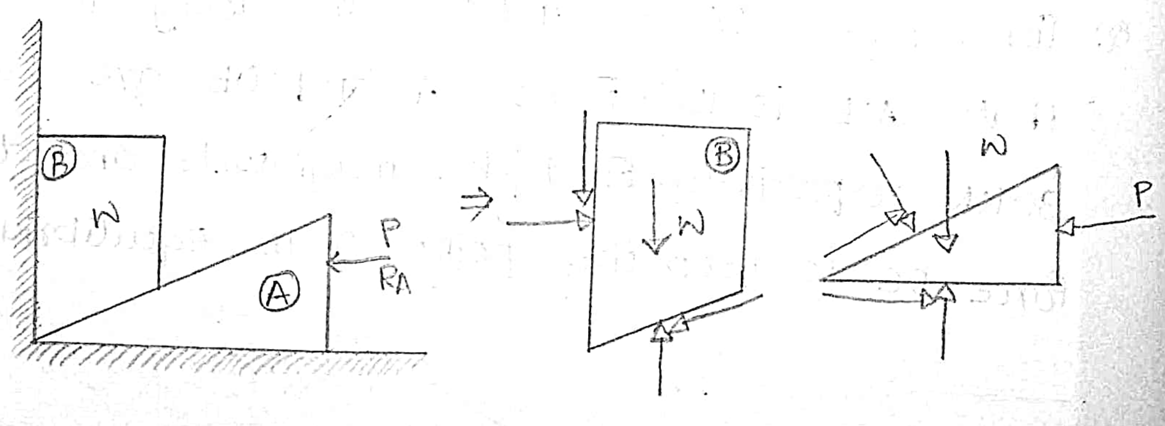
17/8/2019

Free Body Diagram (FBD)

Free body diagram is a diagram in which a rigid body is isolated from the system and all active forces applied to the body and reactive forces as a result of mechanic contact are represented.

e.g.,

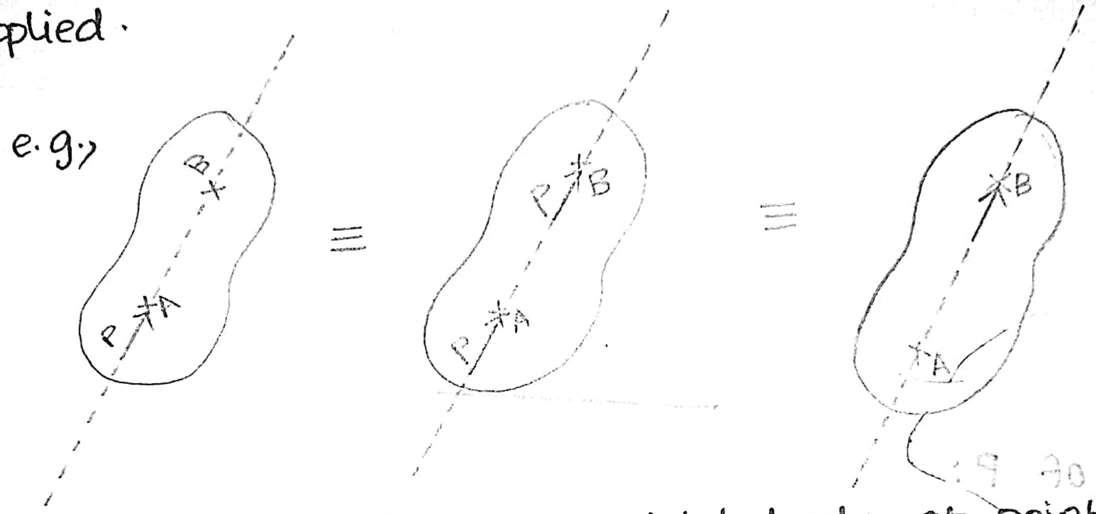




Principle of Transmissibility of Forces

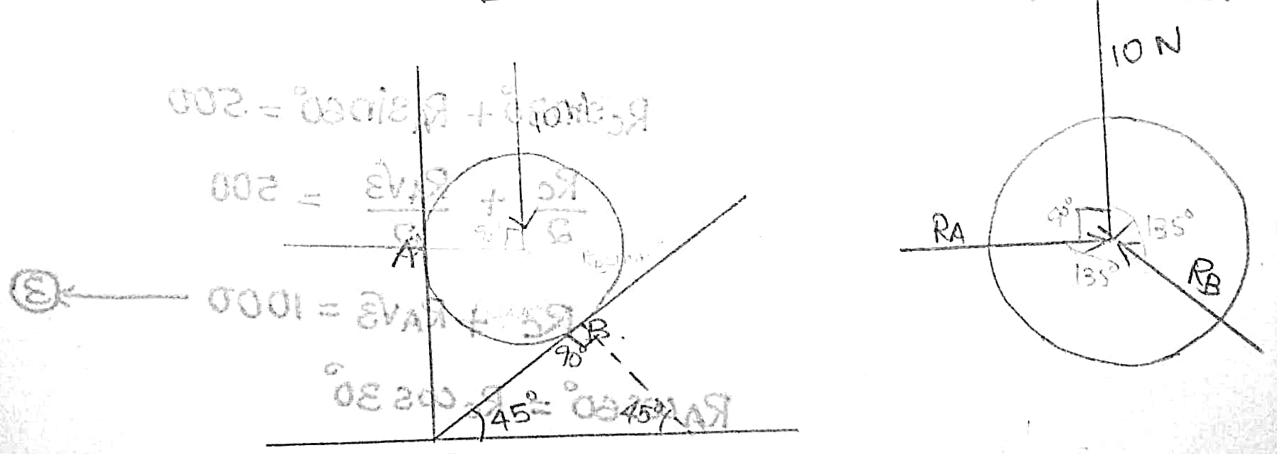
It states that the point of application of a force can be transmitted along its line of action without changing the effect of force on any rigid body to which it is applied.

e.g.,



The force P acts on a rigid body at point A. According to the principle of transmissibility of forces, this force has the same effect on the body as a force P applied at point B along its line of action.

Q: A steel ball rests in a groove, the sides of which are smooth. One side of groove is vertical and the other inclined at 45° to the horizontal. The weight of the ball is 10N. Find the reactions at A and B points of contact.



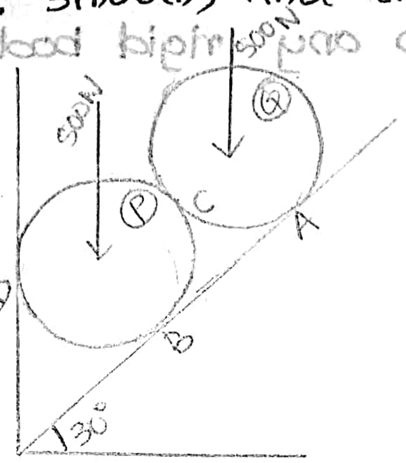
135
135
270
90
360

$$\frac{R_A}{\sin 45^\circ} = \frac{R_B}{\sin 90^\circ} = \frac{10}{\sin 45^\circ}$$

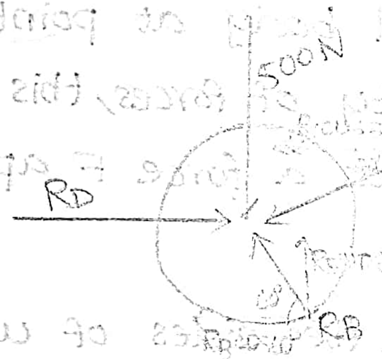
$R_A = 10\,N$

$R_B = \frac{10}{\frac{1}{\sqrt{2}}} = \underline{\underline{10\sqrt{2}\,N}}$

Q: Two identical balls each of weight 500 N are supported by an inclined plane and vertical wall. Assuming the surfaces to be smooth, find the reactions at A, B, C and D.



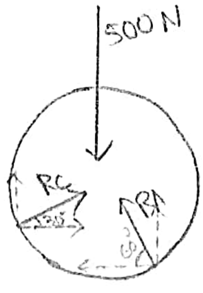
FBD of P :



$$R_B \cos 60^\circ + R_C \cos 30^\circ - R_D = 0 \longrightarrow (1)$$

$$R_C \sin 30^\circ + 500 - R_B \sin 60^\circ = 0 \longrightarrow (2)$$

FBD of Q :



$$R_C \sin 30^\circ + R_A \sin 60^\circ = 500$$

$$\frac{R_C}{2} + \frac{R_A\sqrt{3}}{2} = 500$$

$$R_C + R_A\sqrt{3} = 1000 \longrightarrow (3)$$

$$R_A \cos 60^\circ = R_C \cos 30^\circ$$

$$\frac{R_A}{2} = \frac{R_C\sqrt{3}}{2}$$

$$R_A = R_C\sqrt{3}$$

Sub. in (3)

$$R_C + 3R_C = 1000$$

$$4R_C = 1000$$

$$\underline{\underline{R_C = 250\text{ N}}}$$

$$R_A = 250\sqrt{3}\text{ N} \approx \underline{\underline{433\text{N}}}$$

Sub. R_C in ②,

$$\frac{250}{2} + 500 = R_B\frac{\sqrt{3}}{2}$$

$$250 + 1000 = R_B\sqrt{3}$$

$$R_B\sqrt{3} = 1250$$

$$R_B = \frac{1250}{\sqrt{3}} = \frac{1250\sqrt{3}}{3} \approx \underline{\underline{721\text{ N}}}$$

① $\Rightarrow$ $$R_D = 250\frac{\sqrt{3}}{2} + \frac{1250\sqrt{3}}{3} \times \frac{1}{2} = 125\sqrt{3} + 208.3\sqrt{3}$$

$$= 333.3\sqrt{3}\text{ N}$$

$$= \underline{\underline{577\text{ N}}}$$

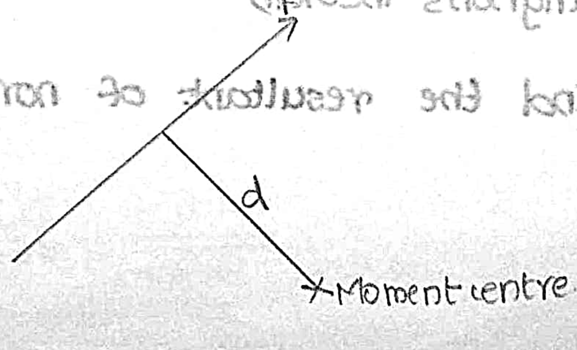
19/8/2019.

MOMENT

Moment of a force about a point or an axis

The tendency of a force to rotate the body in the direction of its application of force about a point that is not on the line of action of the force is called Moment of force or simply moment.

It is the turning effect produced by the force on a rigid body about a point.



Moment, $M = F \times d$

Unit is Nm or kNm

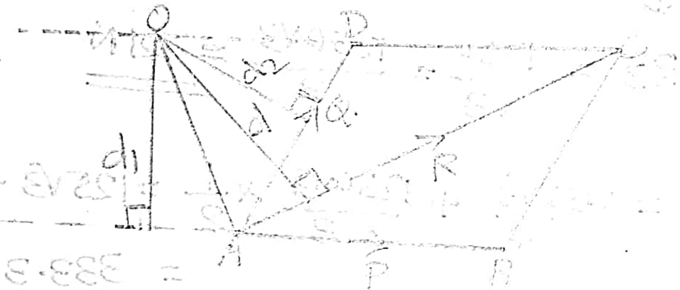
Nature: It can be either clockwise or anticlockwise.

VARIGNON'S THEOREM

It states that the moment of a force about any axis is equal to sum of moments of its components about the same axis.

Proof:

Let P and Q are the components of a force.



Consider ΔOAC,

$$\Delta OAC = \Delta OAD + \Delta DAC$$

$$\Delta DAC = \Delta ABC$$

$$\Delta OAC = \Delta OAD + \Delta ABC$$

$$\frac{1}{2} \times AC \times d = \frac{1}{2} \times AD \times d_2 + \frac{1}{2} \times AB \times d_1$$

$$AC \times d = AD \times d_2 + AB \times d_1$$

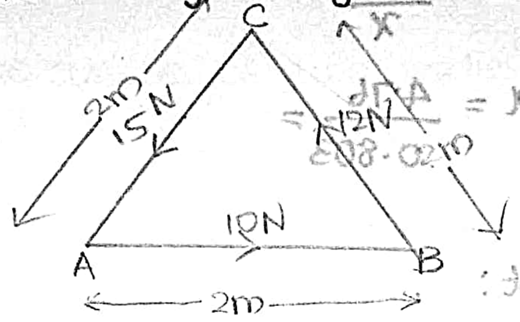
$$R \times d = Q \times d_2 + P \times d_1$$

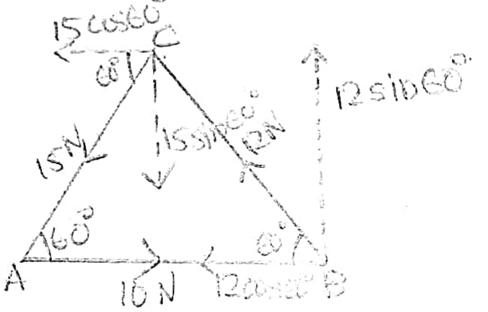
$$Pd_1 + Qd_2 = R \times d$$

Application of Varignon's Theorem

Used to find the resultant of non-concurrent coplanar forces.

Q: Find the resultant of given system of forces.





$\Sigma F_x = 10 - 12\cos 60° - 15\cos 60°$

$= \underline{-3.5 N}$

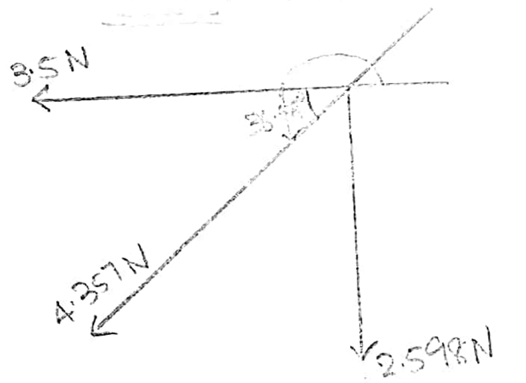
$\Sigma F_y = 12\sin 60° - 15\sin 60°$

$= -1.5\sqrt{3} = \underline{-2.598 N}$

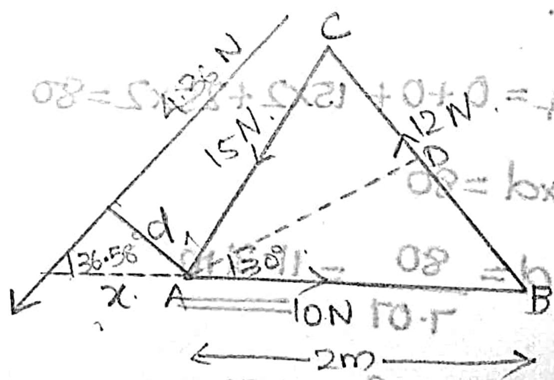
$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{18.999} = \underline{4.357 N}$

$\theta = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x} = \tan^{-1}\frac{2.598}{3.5} = \underline{36.58°}$

$\alpha = 180 + 36.58° = \underline{216.58°}$



- Find the moment of component forces with respect to reference point. Choose A as the reference point.



$\cos 30° = \frac{AD}{AB}$

$AD = \frac{2\sqrt{3}}{2} = \underline{\sqrt{3} m}$

$\Sigma M_A = 0 + 0 + 12\sqrt{3}$

$= \underline{20.76 Nm}$

(Counter clockwise)

$R \times d = 20.76$

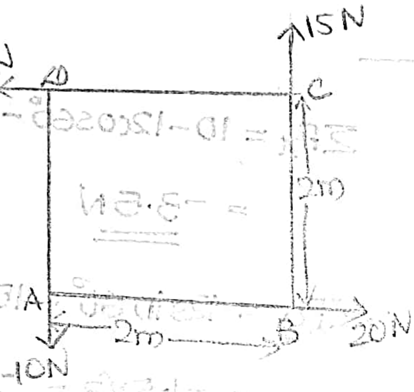
$d = \frac{20.76}{4.36} = \underline{4.76 m}$

$$\cos 36.58^\circ = \frac{4.76}{x}$$

$\sin 36.58^\circ$

$$x = \frac{4.76}{0.803} = 5.927$$

Q: Find the resultant:



$$\Sigma F_x = 20 - 25 = -5\,N$$

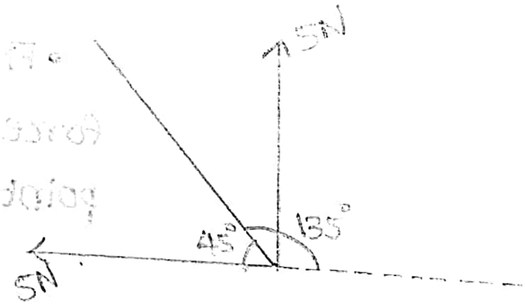
$$\Sigma F_y = 15 - 10 = 5\,N$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{25+25} = \sqrt{50}$$

$$= 5\sqrt{2} = 7.07\,N$$

$$\theta = \tan^{-1}\frac{5}{5} = \tan^{-1} 1 = 45^\circ$$

$$\alpha = 135^\circ$$

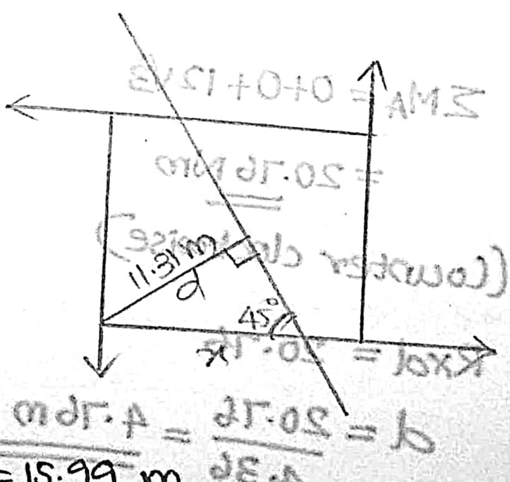


Let A be the reference point.

$$\Sigma M_A = 0 + 0 + 15\times2 + 25\times2 = 80$$

$$R \times d = 80$$

$$d = \frac{80}{7.07} = 11.31\,m$$

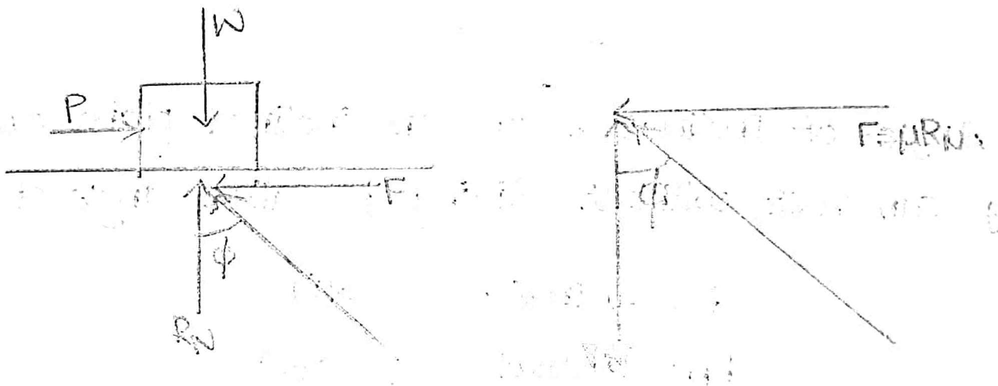


$$\sin 45^\circ = \frac{11.31}{x}$$

$$x = \frac{11.31}{1/\sqrt{2}} = 11.31\sqrt{2} = 15.99\,m$$

FRICTION: MODULE-2.

Whenever the surfaces of two bodies are in contact, there is some resistance to sliding between them. The opposing force to the movement is called friction or force of friction.



Limiting Friction:

Maximum frictional force developed on a surface.

There is a limit beyond which the magnitude of this force cannot increase. If the applied force is more than this limit, there will be movement of one body over the other. This limiting value of frictional force when the motion is impending, it is known as Limiting Friction.

$F \propto R_N$ (R_N – Normal reaction)

$F = \mu R_N$

Coefficient Of Friction

The magnitude of limiting friction bears a constant ratio to the normal reaction between two surfaces. This ratio is called coefficient of friction.

μ_s – Coefficient of static friction

μ_d – coefficient of dynamic friction

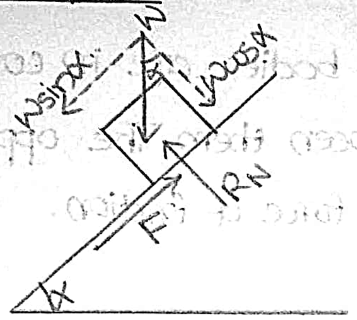
Angle of Friction (ϕ):

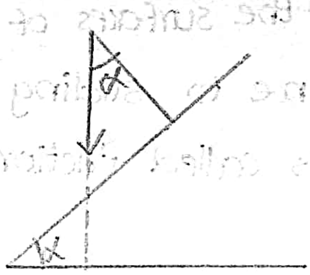
Angle between the resultant and the normal reaction.

$$\tan\phi = \frac{F}{R_N} = \frac{\mu R_N}{R_N} = \mu$$

$$\tan\phi = \mu$$

Angle of Repose :





Angle of inclination of an inclined plane on which a body can rest without sliding is called angle of repose

$$F = W\sin\alpha \longrightarrow (1)$$

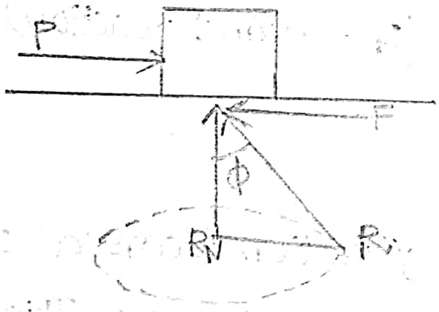
$$R_N = W\cos\alpha \longrightarrow (2)$$

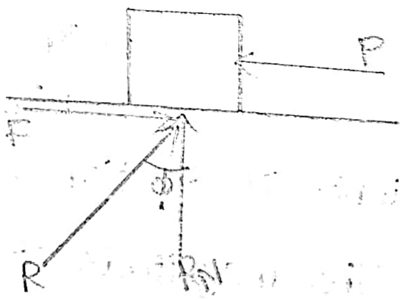
$$\frac{(1)}{(2)} \Rightarrow \tan\alpha = \frac{F}{R_N} = \frac{\mu R_N}{R_N}$$

$$\tan\alpha = \mu = \tan\phi$$

$$\underline{\underline{\alpha = \phi}}$$

Cone of Friction:

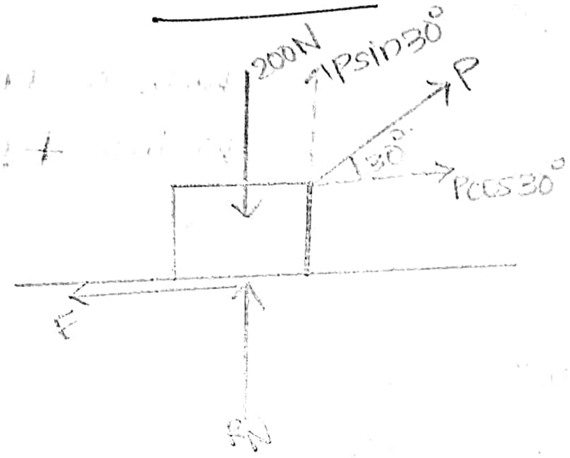




It is defined as the right circular cone with vertex at the point of contact of the two surfaces, axis in the direction of normal reaction N and semi-vertical angle equal to angle of friction.

Base radius of the cone will be frictional force.

Q: A block of weight 200N is placed on a rough horizontal floor. If μ is 0.25, find the pull P required to move the block, if P is inclined upwards at 30° to horizontal.



$$F = \mu R_N = 0.25 R_N$$

$$P\sin 30° + R_N = 200 \longrightarrow (1)$$

$$P\cos 30° = F \longrightarrow (2)$$

$$P\cos 30° = 0.25 R_N$$

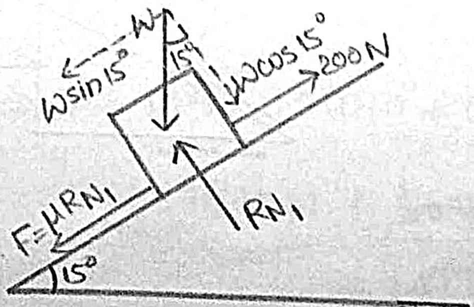
$$(1) \Rightarrow \frac{P}{2} + \frac{P\sqrt{3}}{2\times 0.25} = 200$$

$$0.5 + 3.46P = 200$$

$$P = \frac{200}{3.96} = 50.45\,N$$

Q: A force of 200N is required to move a body up an inclined plane of angle 15° when the force is applied parallel to the plane. When the inclination of the plane is made 20°, force required, again applied parallel to plane is found to be 230N. Find weight and coefficient of friction.

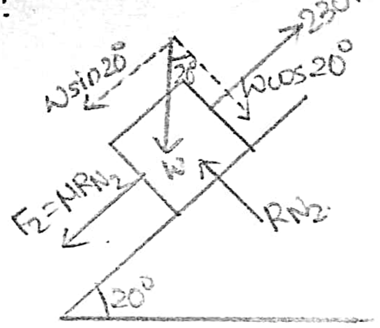
Case : 1



$W\cos 15^\circ = RN_1$

$W\sin 15^\circ + \mu RN_1 = 200$

Case-2 :



$W\cos 20^\circ = RN_2$

$W\sin 20^\circ + \mu RN_2 = 230$

$W\sin 15^\circ + \mu W\cos 15^\circ = 200$

$W\sin 20^\circ + \mu W\cos 20^\circ = 230$

$$\frac{W(0.26 + 0.96\mu)}{W(0.34 + 0.94\mu)} = \frac{200}{230}$$

$0.26\times 230 + 0.96\mu \times 230 = 200\times 0.34 + 200\times 0.94\mu$

$59.8 + 220.8\mu = 68 + 188\mu$

$32.8\mu = 8.2$

$\mu = 0.26$

$220.8 - 188 = 32.8$

$68.0 - 59.8 = 8.2$

$0.26W + 0.96\times 0.26W = 200$

$1.96W = \frac{200}{0.26}$

$W = 392.46N$

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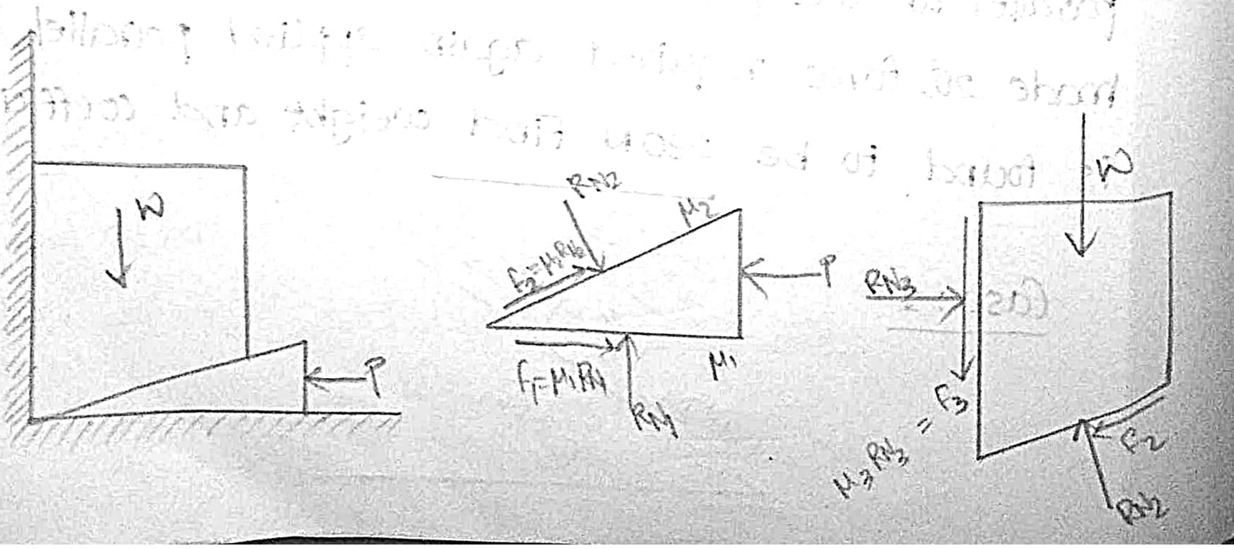
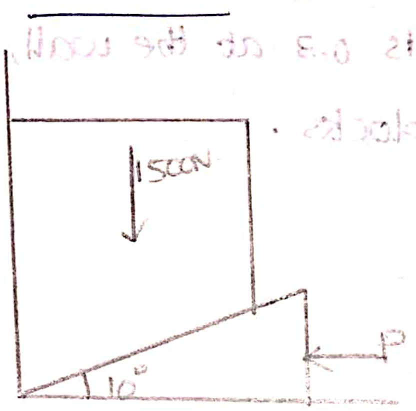
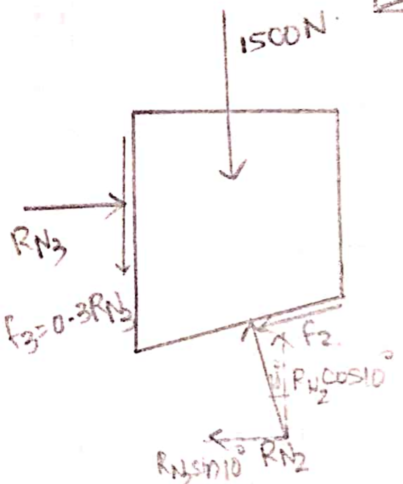
Wedge Friction



Q: Find the horizontal force P to be applied on a 10° wedge which weighs 1500 N load. The coefficient of friction is 0.3 at all contact surface.





$$R_{N_2}\cos 10° - 1500 - 0.3R_{N_2}\sin 10° - 0.3R_{N_3} = 0 \longrightarrow (1)$$

$$R_{N_3} - 0.3R_{N_2}\cos 10° - R_{N_2}\sin 10° = 0 \longrightarrow (2)$$

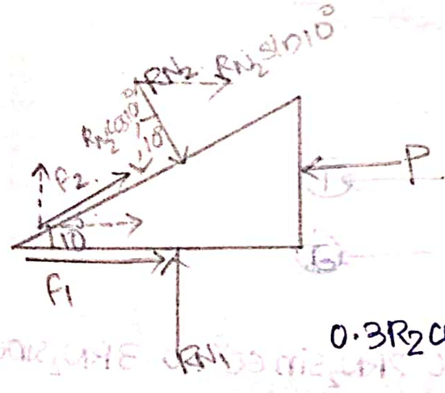
$$0.3 \times (2) \Rightarrow 0.3R_{N_3} - 0.09R_{N_2}\cos 10° - 0.3R_{N_2}\sin 10° = 0.$$

$$(1) + (2) \Rightarrow 0.91R_{N_2}\cos 10° - 0.6R_{N_2}\sin 10° = 1500$$

$$0.896R_{N_2} - 0.104R_{N_2} = 1500$$

$$0.792R_{N_2} = 1500$$

$$R_{N_2} = 1893.9\text{ N}$$



$$f_2\sin 10° + R_{N_1} = R_{N_2}\cos 10°$$

$$R_{N_2}\sin 10° + f_1 = P$$

$$0.3R_{N_2}\sin 10° + R_{N_1} = R_{N_2}\cos 10° \longrightarrow (3)$$

$$0.3R_2\cos 10° + R_{N_2}\sin 10° + 0.3R_{N_1} = P \longrightarrow (4)$$

Sub. R_{N_2} in (3),

$$0.052 \times 1893.9 + R_{N_1} = 0.984R_{N_2}$$

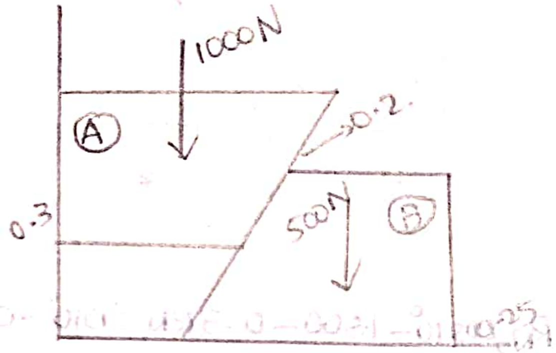
$$R_{N_1} = 1765.114\text{ N}$$

$$(4) \Rightarrow P = 1893.9\sin 10° + 0.3 \times 1765.114 + 0.3 \times 1893.9\cos 10°$$

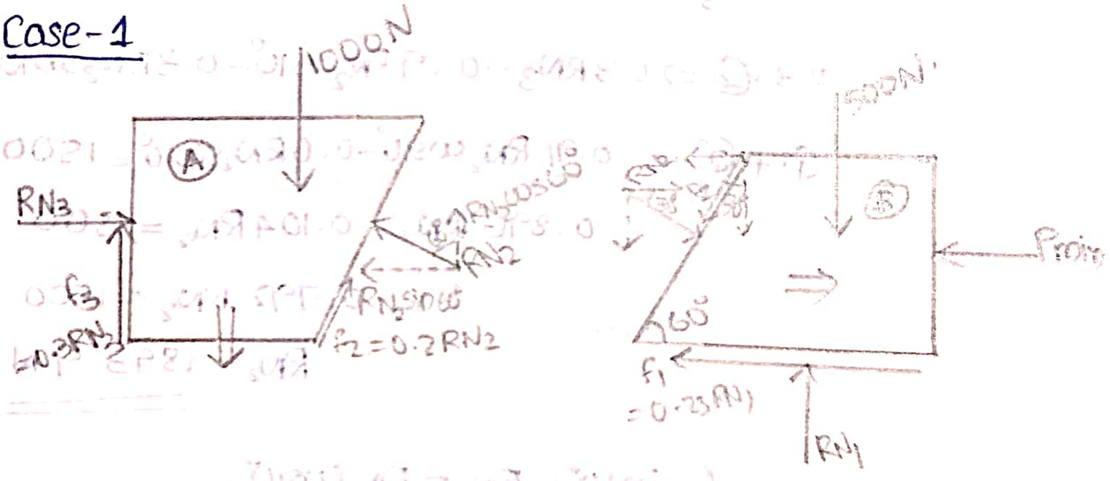
$$= 858.4\text{ N} + 559.54$$

$$= 1417.94\text{ N}$$

Q: Two blocks A and B are resting against a wall and a floor as shown in figure. Find the range of value of force P applied to the lower block B for which the system remains in equilibrium. Coefficient of friction is 0.3 at the wall, 0.25 at the floor and 0.2 between the blocks.



Case-1



For A: $RN_2 \cos 60^\circ + f_2 \sin 60^\circ + f_3 - 1000 = 0 \longrightarrow (1)$

$f_2 \cos 60^\circ - RN_2 \sin 60^\circ + RN_3 = 0 \longrightarrow (2)$

$(1) - (2) \Rightarrow RN_2 \cos 60^\circ - 0.06 RN_2 \cos 60^\circ + 0.2 RN_2 \sin 60^\circ + 0.3 RN_2 \sin 60^\circ - 1000 = 0$

$0.903\, RN_2 = 1000$

$RN_2 = \frac{1000}{0.903} = 1107.4\,N$

For B:

$RN_1 - 500 - f_2 \cos 30^\circ - RN_2 \sin 30^\circ = 0 \longrightarrow (3)$

$RN_2 \cos 30^\circ - f_2 \sin 30^\circ - f_1 - P_{min} = 0 \longrightarrow (4)$

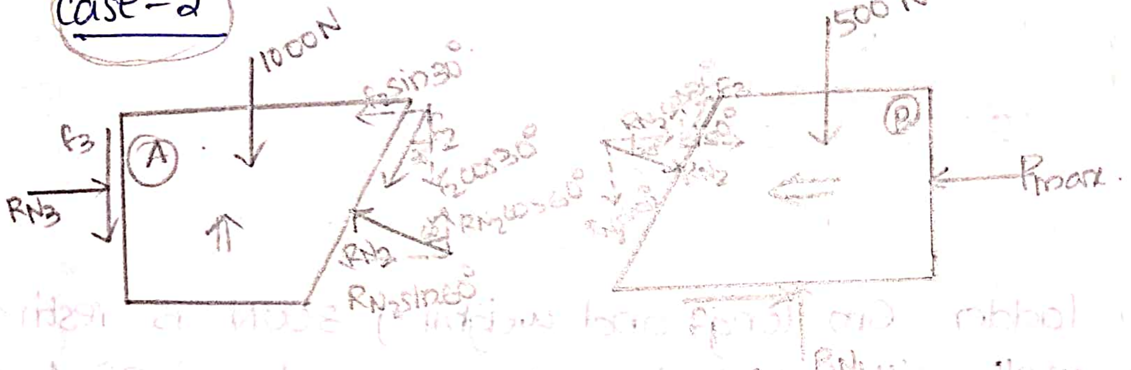
$③ \Rightarrow R_{N_1} - 500 - 0.2 \times 1107.4 \cos 30^\circ - 1107.4 \sin 30^\circ = 0$

$$R_{N_1} = 1245.5 \text{ N}$$

$④ \Rightarrow P_{min} = 1107.4 \cos 30^\circ - 0.2 \times 1107.4 \sin 30^\circ - 0.25 \times 1245.5$

$$= 536.92 \text{ N}$$

Case-2



For A: $R_{N_2} \cos 60^\circ - f_2 \cos 30^\circ - f_3 - 1000 = 0 \longrightarrow (A)$

$R_{N_3} - f_2 \sin 30^\circ - R_{N_2} \sin 60^\circ = 0 \longrightarrow (B)$

$R_{N_2} \cos 60^\circ - 0.2 R_{N_2} \cos 30^\circ - 0.3 R_{N_3} - 1000 = 0$

$0.3 R_{N_3} - 0.2 \times 0.3 R_{N_2} \sin 30^\circ - 0.3 R_{N_2} \sin 60^\circ = 0$

$(+) \Rightarrow R_{N_2} \cos 60^\circ - 0.2 R_{N_2} \cos 30^\circ - 0.06 R_{N_2} \sin 30^\circ - 0.3 R_{N_2} \sin 60^\circ = 1000$

$$0.0369 R_{N_2} = 1000$$

$$\cancel{R_{N_2} = 27100.27 \text{N}}$$

$$R_{N_2} = 27{,}041.64$$

For B: $f_2 \sin 60^\circ - R_{N_2} \sin 30^\circ - 500 + R_{N_1} = 0 \longrightarrow (C)$

$f_2 \cos 60^\circ + R_{N_2} \cos 30^\circ + f_1 - P_{max} = 0 \longrightarrow (D)$

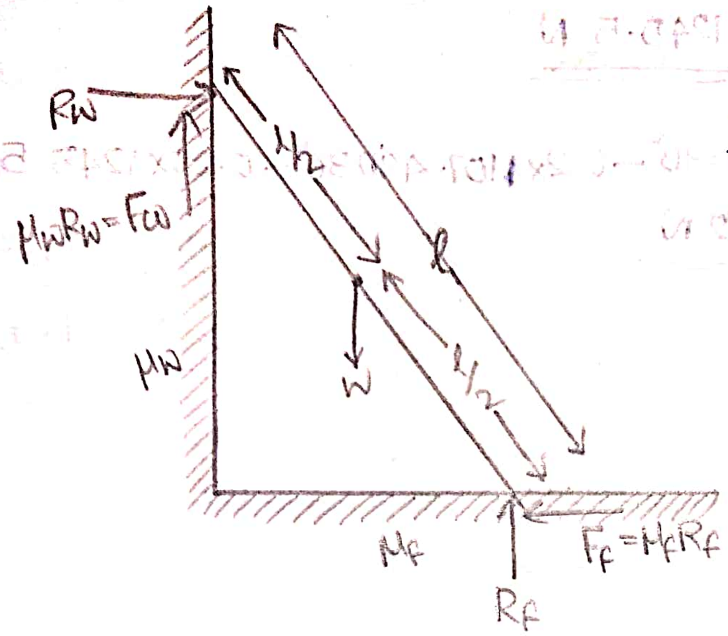
$(C) \Rightarrow R_{N_1} = 500 + 27{,}041.64 \sin 30^\circ - 0.2 \times 27{,}041.64 \cos 60^\circ$ 1584

$$= 11{,}316.65 \text{N}$$

$(D) \Rightarrow P_{max} = 0.2 \times 27{,}041.64 \cos 60^\circ + 27{,}041.64 \cos 30^\circ + 0.25 \times 11{,}316.65$

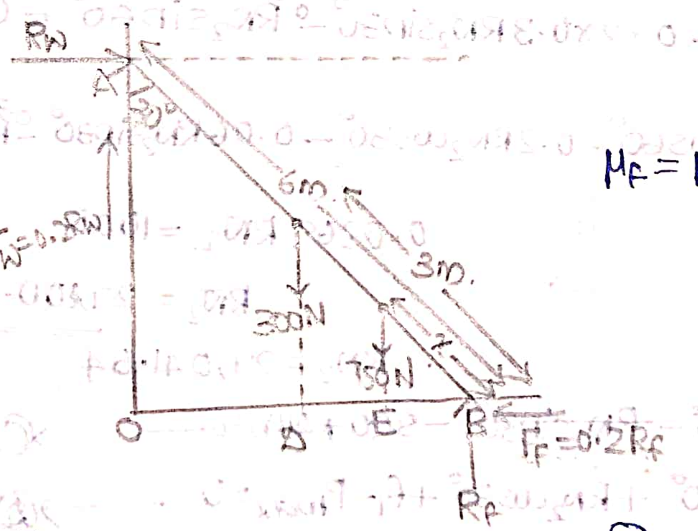
$$= 28952.07 \text{ N}$$

Ladder Friction



$\Sigma F_x = 0$
$\Sigma F_y = 0$

Q: A uniform ladder 6m long and weighing 300N is resting against a wall with which it makes an angle of 30°. A man weighing 750N climbs up the ladder. At what position along the ladder from the bottom end does the ladder slip? The coefficient of friction of ladder with respect to wall and floor is 0.2.



$\mu_F = \mu_W = 0.2$.

$\Sigma F_x = 0 \qquad R_W - 0.2R_F = 0 \longrightarrow ①$

$\Sigma F_y = 0 \qquad 0.2R_W - 300 - 750 + R_F = 0 \longrightarrow ②$

$R_W = 0.2R_F$.

$0.2(0.2R_F) - 1050 + R_F = 0$

$0.04R_F + R_F = 1050$

$1.04R_F = 1050$

$R_F = \frac{1050}{1.04} = 1009.6N$

$R_w = 0.2 \times 1009.6 = \underline{201.92N}$

For static equilibrium, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$

$$\Sigma M_B = R_w \times OA + 0.2 R_w \times OB - 300 \times BD - 750 \times BE = 0$$

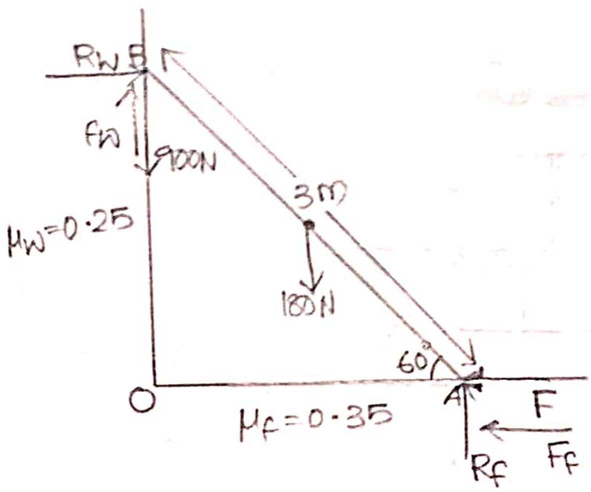
$$201.92 \times 6\cos 30° + 0.2 \times 201.92 \times 6 \sin 30° - 300 \times 3 \sin 30° - 750 \times x \sin 30° = 0$$

$$720.35 = \frac{750 \times x}{2}$$

$$x = \frac{720.35}{375} = \underline{1.92m}$$

22/8/2019

Q: A 3m long ladder weighing 180N is placed against a wall. Ladder is inclined 60° to floor. μ_w is 0.25 and μ_f is 0.35. In addition to its self weight it supports a man weighing 900N at the top end B. Find the minimum horizontal force to be applied at the floor level to prevent slipping. (KTU May 2019)



$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$.

$$F_w - 180 - 900 + R_f = 0$$

$$0.25 R_w - 1080 + R_f = 0 \longrightarrow (1)$$

$$R_w - F_f - F = 0$$

$$R_w - 0.35 R_f - F = 0 \longrightarrow (2)$$

$$\Sigma M_A = 0$$

$$R_W \times OB + F_W \times OA - 900 \times OA - 180 \times AC = 0$$

$$R_W \times 3\sin 60^\circ + 0.25 R_W \times 3\cos 60^\circ - 900 \times 3\cos 60^\circ - 180 \times \frac{3}{2}\cos 60^\circ = 0$$

$\sin 60^\circ = \frac{OB}{3}$

$\cos 60^\circ = \frac{OA}{3}$

$\cos 60^\circ = \frac{AC}{3/2}$

$$2.97 R_W - 1485 = 0$$

$$\underline{R_W = 500\,N}$$

$$0.25 \times 500 - 1080 + R_F = 0$$

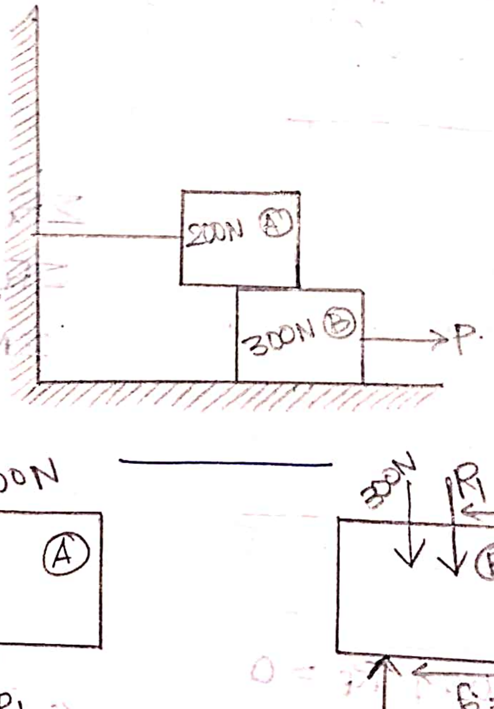
$$\underline{R_F = 955\,N}$$

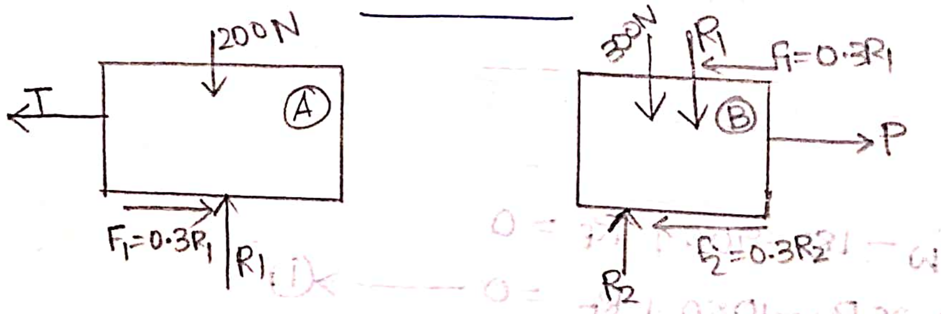
$$F = 500 - 0.35 \times 955$$

$$\underline{= 165.75\,N}$$

Friction Between Connected Bodies

Q: Find force P required to move block B. Coefficient of friction is 0.3 for all surfaces.





For A: $R_1 - 200 = 0$

$$R_1 = 200\,N$$

$$0.3 R_1 - T = 0$$

$$T = 0.3 \times 200$$

$$\underline{= 60\,N}$$

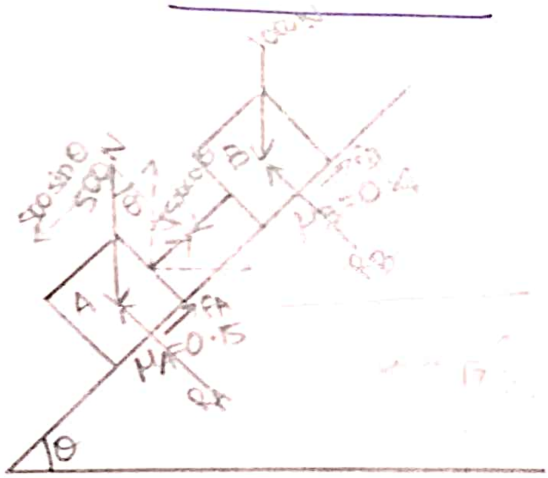
For B: $P - 0.3R_1 - 0.3R_2 = 0$

$$P = 0.3R_1 + 0.3R_2 \longrightarrow ①$$

$$R_2 = R_1 + 300 = 200 + 300 = 500N$$

$$① \Rightarrow P = 60 + 150 = 210N$$

Q: Two blocks A and B weighs 500N and 1000N are placed on an inclined plane. The blocks are connected by a string parallel to inclined plane. Coefficient of friction at point A is 0.15 and at point B is 0.4. Find the inclination of the plane when the blocks start moving. Also calculate tension in the string.



For A: $f_A - 500\sin\theta = T$

$$0.15R_A - 500\sin\theta = T$$

$$R_A = 500\cos\theta$$

$$T + 75\cos\theta - 500\sin\theta = 0 \longrightarrow ①$$

For B: $-T - 1000\sin\theta + f_B = 0$

$$-T - 1000\sin\theta + 0.4R_B = 0$$

$$R_B = 1000\cos\theta$$

$$-T - 1000\sin\theta + 400\cos\theta = 0 \longrightarrow ②$$

$$① + ② \Rightarrow 475\cos\theta - 1500\sin\theta = 0$$

$$475\cos\theta = 1500\sin\theta$$

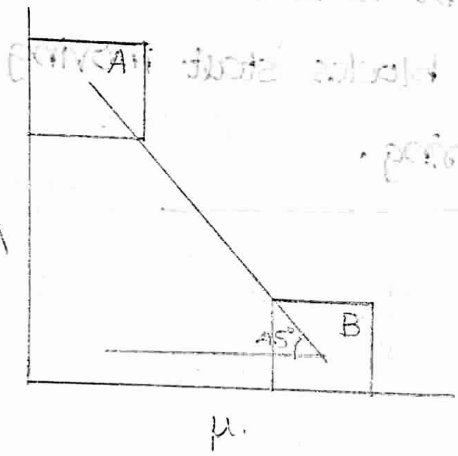
$$\tan\theta = \frac{475}{1500}$$

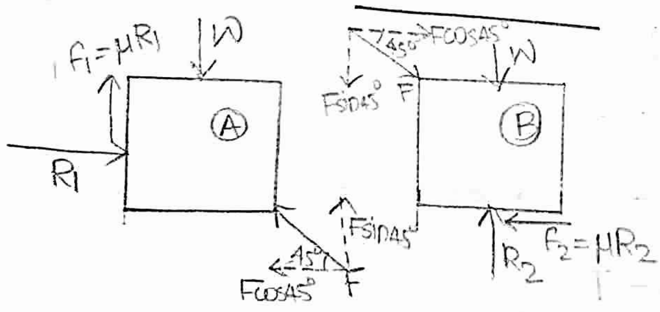
$$\theta = \tan^{-1}(0.316) = 17.57^\circ$$

$$T = 75\cos\theta + 500\sin\theta = 79.43\text{ N}$$

27/8/2019

Q: Two identical blocks A and B of weight W are supported by a rigid bar inclined 45° with horizontal. If both the blocks are in limiting equilibrium, find the coefficient of friction assuming it will be same at floor and wall.





F-compressive force.

For A: $R_1 = F\cos 45^\circ$

$\mu R_1 + F\sin 45^\circ - W = 0$

$\mu F\cos 45^\circ + F\sin 45^\circ = W$

For B: $\mu R_2 = F\cos 45^\circ$

$R_2 - W - F\sin 45^\circ = 0$

$\frac{F}{\mu}\cos 45^\circ - F\sin 45^\circ = W$

$$\mu F\cos 45^\circ + F\sin 45^\circ = \frac{F}{\mu}\cos 45^\circ - F\sin 45^\circ$$

$$\mu + 1 = \frac{1}{\mu} - 1$$

$$\mu - \frac{1}{\mu} = -2$$

$$\mu^2 + 2\mu - 1 = 0$$

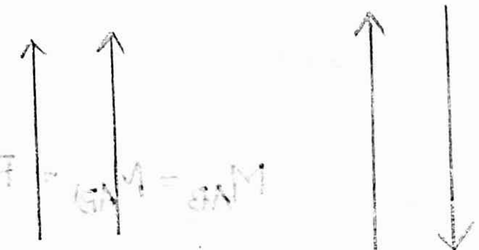
$$\mu = \frac{-2 \pm \sqrt{4+4}}{2}$$

$$= -1 \pm \sqrt{2}$$

$$= -1 + 1.414$$

$$= 0.414$$

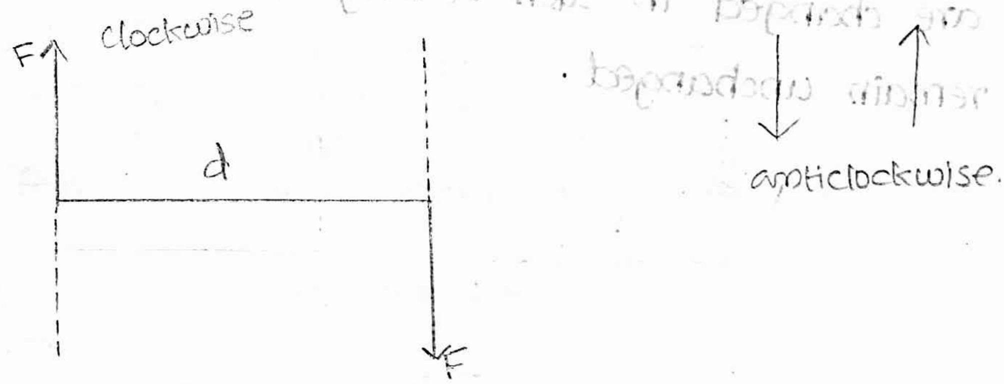
Parallel Coplanar Forces



Like parallel forces   Unlike parallel forces

Couple:

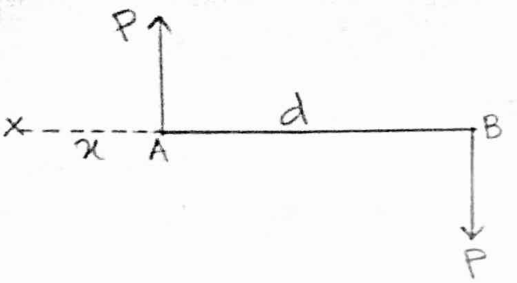
Unlike parallel forces of equal magnitude constitute couple.



Moment of a couple = Fd

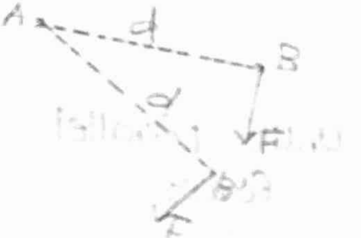
Properties of Force Couple

1. The two forces constituting a couple are equal in magnitude and opposite in direction, therefore, resultant is equal to zero.
2. Moment of a couple about any point in the plane of couple is a constant and independent of the moment centre.



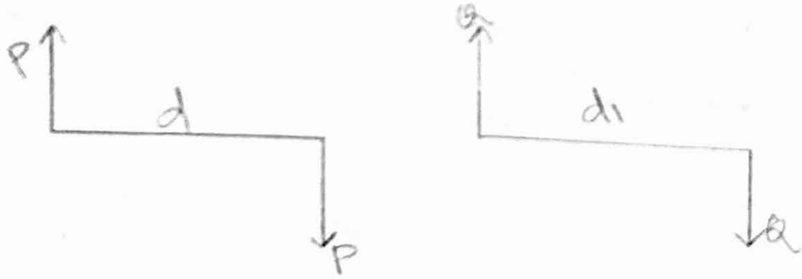
$$M = -Px + P(x+d)$$
$$= -Px + Px + Pd$$
$$= \underline{\underline{Pd}}$$

3. Action of a couple on a rigid body will not be changed if its arm is turned in the plane of couple through any angle about one of its ends.



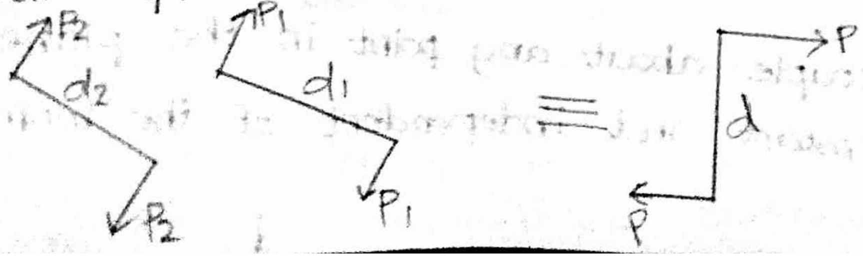
$$M_{AB} = M_{AB'} = F \times d$$

4. The action of a couple on a body does not change if both the magnitudes of the forces and the arm of couple are changed in such a way that the moment of couple remain unchanged.



$$P \times d = Q \times d_1$$

5. Several couples in one plane can be replaced by a single couple acting in the same plane such that moment of single couple is equal to algebraic sum of moments of given couples.



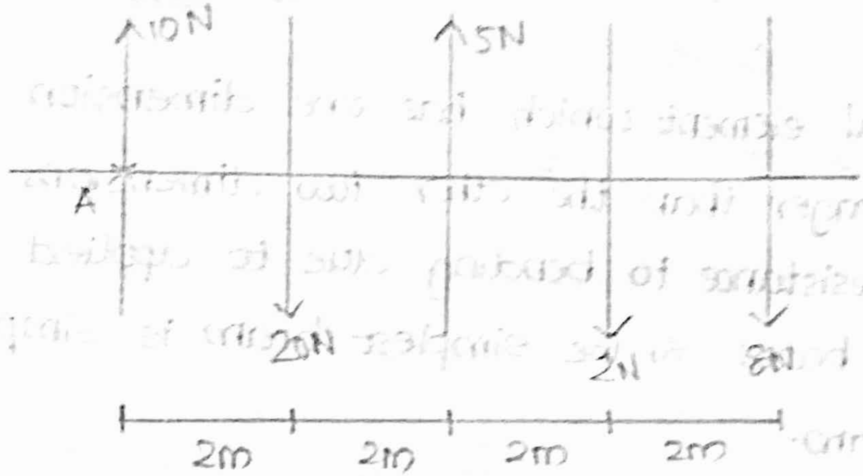
$$P_1 d_1 + P_2 d_2 = Pd$$

6. A given couple can be resolved into several component couples by choosing the component couples in such a manner that the algebraic sum of their moments is equal to the moment of the given couple.

3/8/2019

Resultant Of Parallel Forces

Q: Find the resultant of parallel forces.



$\Sigma F_x = 0$

$\Sigma F_y = 10 - 20 + 5 - 2 - 8 = -15N$

Resultant force is 15N in downward direction.

$\Sigma M_A = 20\times2 - 5\times4 + 2\times6 + 8\times8$

$= 40 - 20 + 12 + 64$

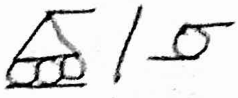
$= 96\ Nm$

$Rx = 96$

$x = \frac{96}{15} = 6.4\ m$ (Position of force).

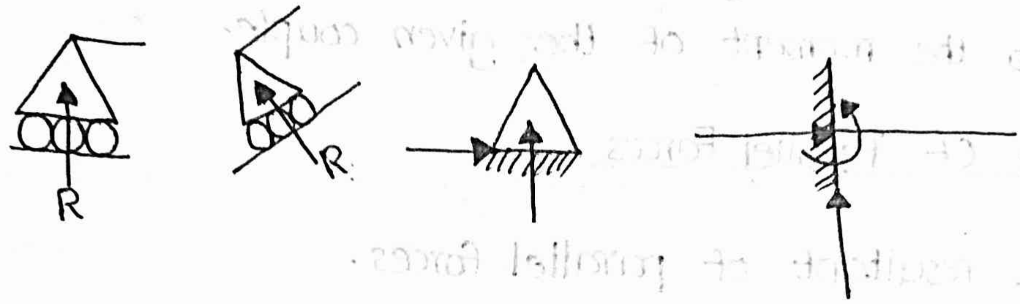
SUPPORT REACTIONS.

Types of supports

1. Roller 

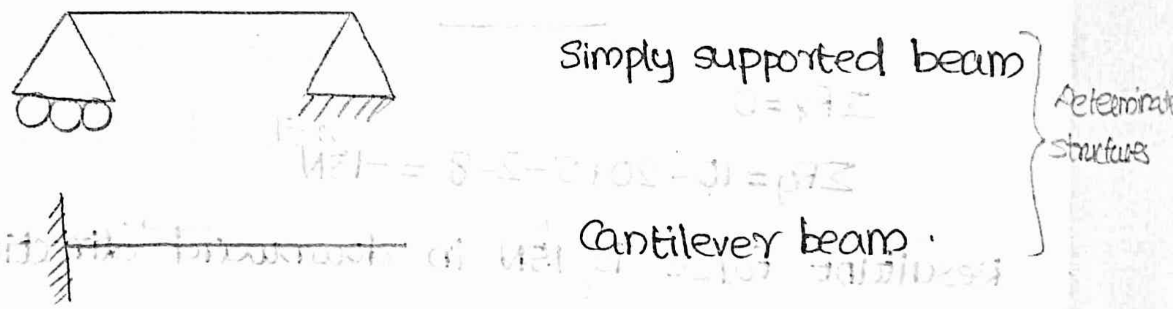
2. Hinged support

3. Fixed



BEAM:

A structural element which has one dimension considerably larger than the other two dimensions and which offers resistance to bending due to applied loads is known as beam. The simplest beam is simply supported beam.

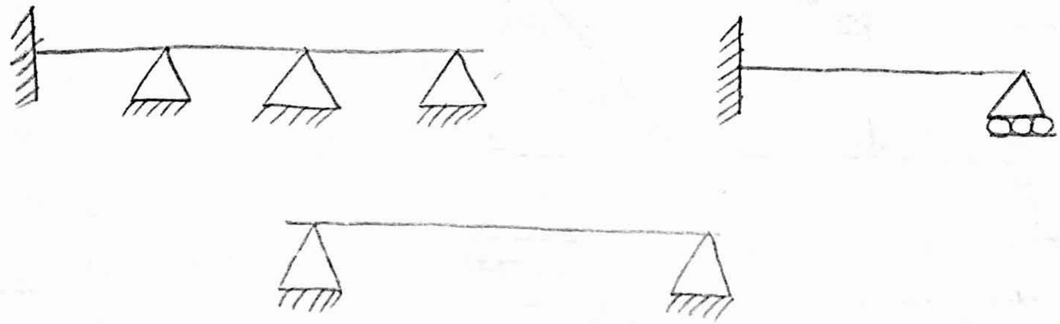


Determinate structures are analysed just by the use of basic equilibrium equations,

$$\Sigma H = 0 \quad , \quad \Sigma V = 0 \quad , \quad \Sigma M = 0$$

Indeterminate structures are not capable of being analysed by mere use of basic equilibrium equations. Along with the basic equilibrium equations, some extra conditions are required to be used like compatibility conditions of deformations.

Examples of indeterminate structures.



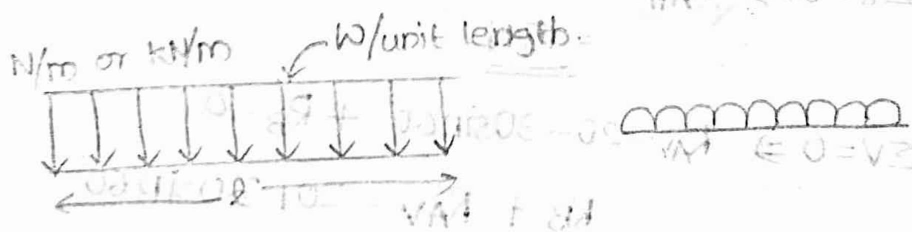
DIFFERENT TYPES OF LOADS ON BEAMS.

1. Point load / concentrated load:

Act at a point.

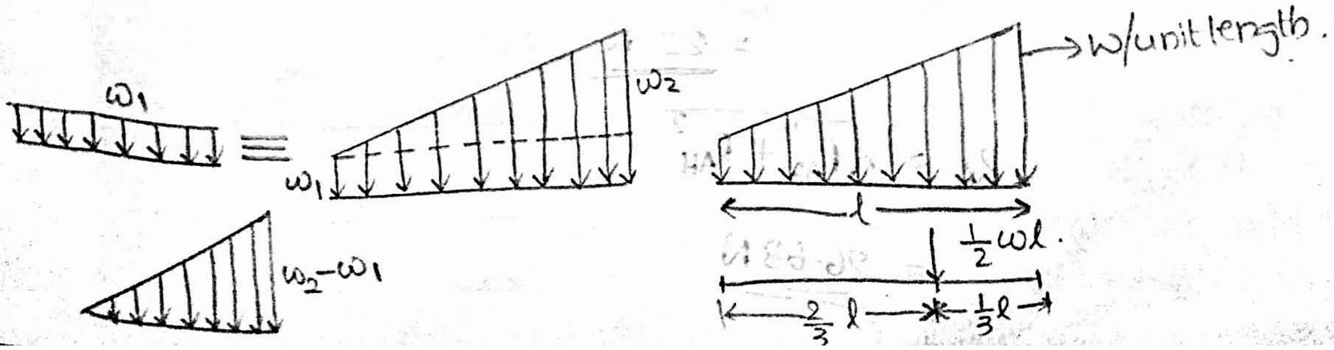
2. Uniformly distributed load (udl):

Load spread along the length of the beam.

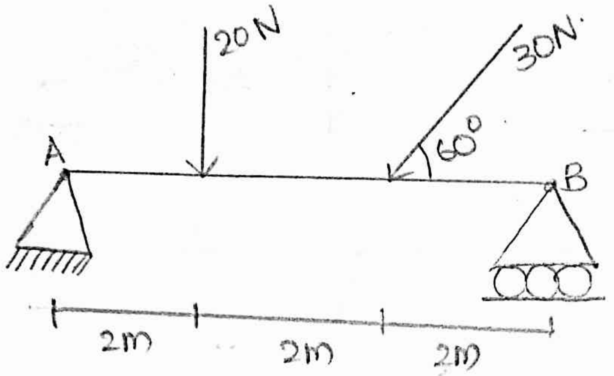


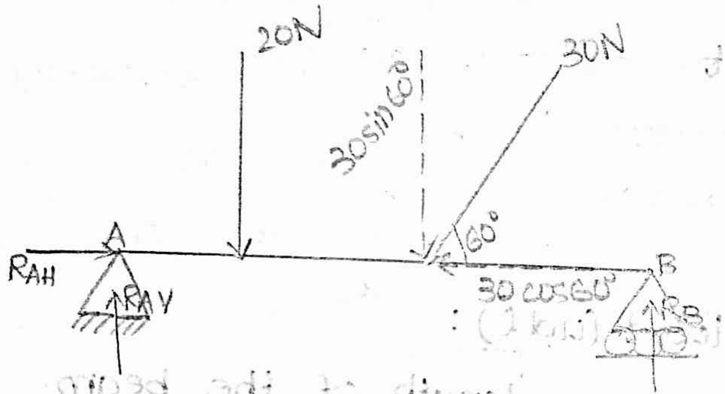
3. Uniformly varying load (uvl):

Load spread along the length of the beam in such a way that rate of loading vary from point to point throughout the distribution length of the beam.



Q: Find the reactions at supports A and B.





$$\Sigma H = 0 \Rightarrow R_{AH} = 30\cos 60^\circ$$

$$= \underline{\underline{15\,N}}$$

$$\Sigma V = 0 \Rightarrow R_{AV} - 20 - 30\sin 60^\circ + R_B = 0$$

$$R_B + R_{AV} = 20 + 30\sin 60^\circ$$

$$R_B + R_{AV} = 45.98$$

$$\Sigma M = 0 \Rightarrow 20 \times 2 - R_B \times 6 + 30\sin 60^\circ \times 4 = 0$$

$$R_B = 23.99\,N$$

$$R_{AV} = 45.98 - 23.99$$

$$= 21.99$$

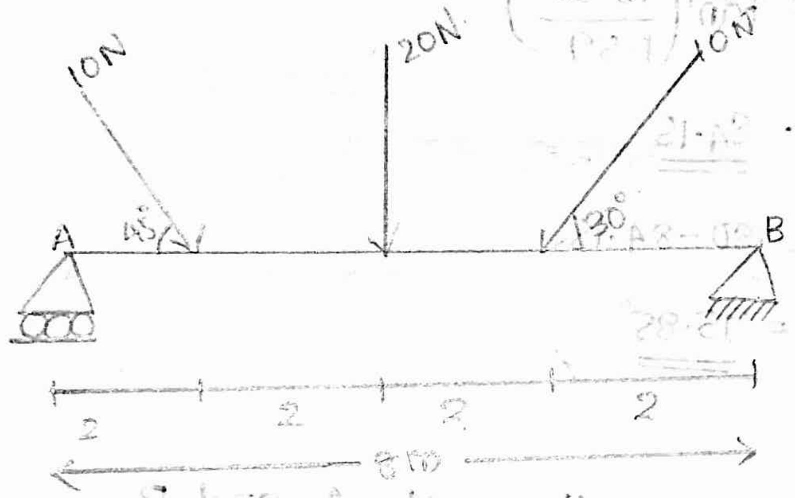
$$= \underline{22\,N}$$

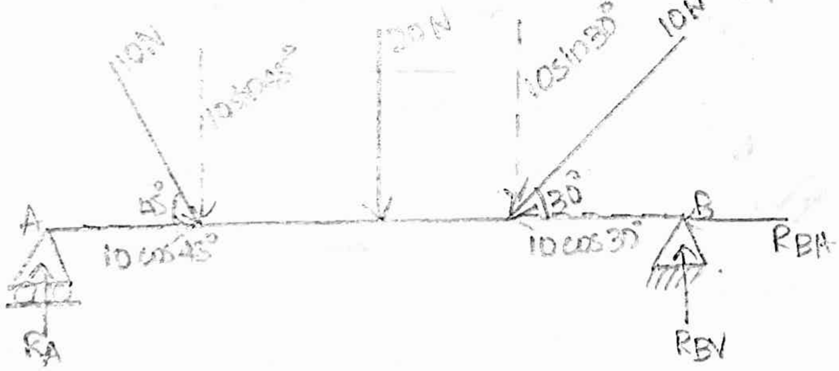
$$R_A = \sqrt{R_{AV}^2 + R_{AH}^2}$$

$$= \underline{\underline{26.63\,N}}$$

$$\theta = \tan^{-1}\left(\frac{R_{AV}}{R_{AH}}\right)$$
$$= \tan^{-1}\left(\frac{22}{15}\right)$$
$$= 55.71^\circ$$

Q: Find the support reactions at A and B.





$$\Sigma H = 0 \Rightarrow 10\cos45^\circ - 10\cos30^\circ - R_{BH} = 0$$
$$R_{BH} = -1.59\text{N}$$

$$\Sigma V = 0 \Rightarrow R_A - 10\sin45^\circ - 20 - 10\sin30^\circ + R_{BV} = 0$$
$$R_A + R_{BV} = 32.07\ \text{N}$$

$$\Sigma M = 0 \Rightarrow 10\sin45^\circ \times 2 + 20\times4 + 10\sin30^\circ\times6 - R_{BV}\times8 = 0$$
$$R_{BV} = 15.52\ \text{N}$$

$$R_A = 32.07 - 15.52$$
$$= 16.55\text{N}$$

$$R_B = \sqrt{(R_{BH})^2 + (R_{BV})^2}$$

$$= \underline{\underline{15.601\ N}}$$

$$\theta = \tan^{-1}\left(\frac{R_{BV}}{R_{BH}}\right)$$

$$= \tan^{-1}\left(\frac{15.52}{1.59}\right)$$

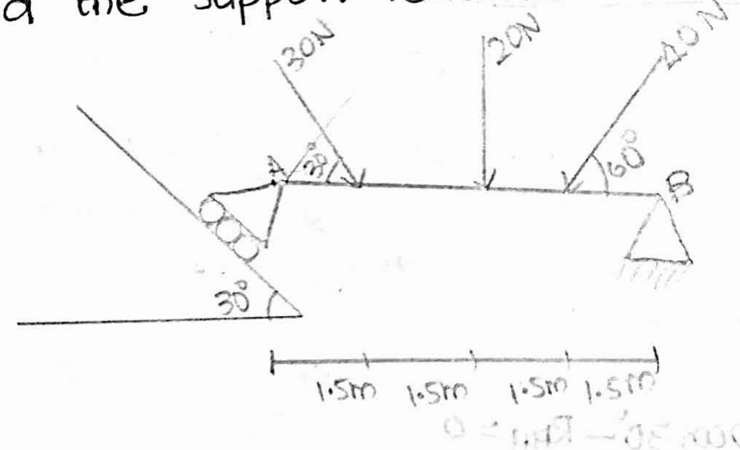
$$= \underline{\underline{84.15^\circ}}$$

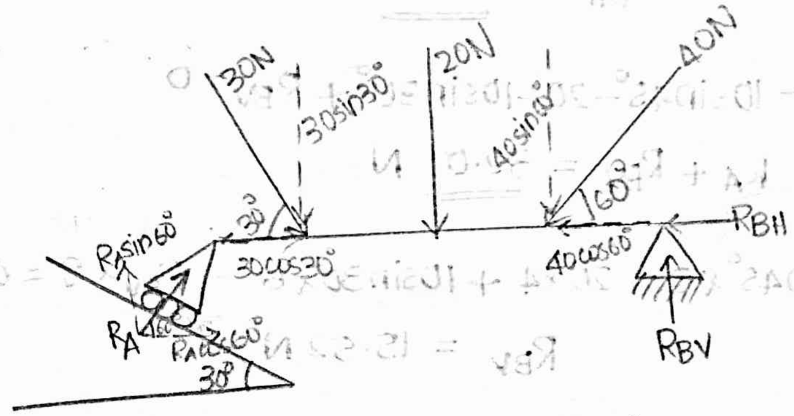
$$\theta_R = 180 - 84.15$$

$$= \underline{\underline{95.85^\circ}}$$

Home Work

Q:1) Find the support reactions at A and B.





$$\Sigma H = 0 \Rightarrow R_A \cos 60^\circ + 30\cos 30^\circ - 40\cos 60^\circ - R_{BH} = 0$$

$$\frac{R_A}{2} - R_{BH} = -5.98 \longrightarrow ①$$

$\Sigma V = 0 \Rightarrow R_A \sin 60^\circ - 30\sin 30^\circ - 20 - 40\sin 60^\circ + R_{BV} = 0$

$$\frac{\sqrt{3} R_A}{2} + R_{BV} = 69.64 \longrightarrow ②$$

$\Sigma M = 0 \Rightarrow 30\sin 30^\circ \times 1.5 + 20 \times 3 + 40\sin 60^\circ \times 4.5 - R_{BV} \times 6 = 0$

$$R_{BV} = \underline{\underline{39.73\ N}}$$

Sub. in ②,

$$\frac{\sqrt{3} R_A}{2} = 29.91$$

$$R_A = \underline{\underline{34.54\ N}}$$

Sub. in ①,

$$R_{BH} = 17.27 + 5.98 = \underline{\underline{23.25\ N}}$$

$$R_B = \sqrt{(R_{BH})^2 + (R_{BV})^2}$$

$$= \sqrt{(23.25)^2 + (39.73)^2}$$

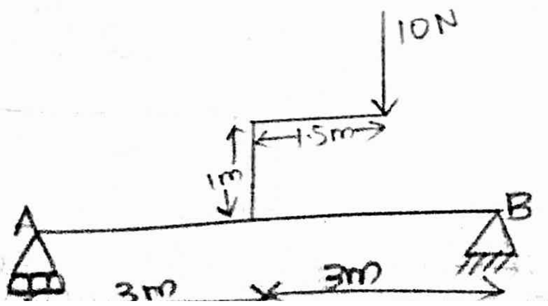
$$= \underline{\underline{46.03\ N}}$$

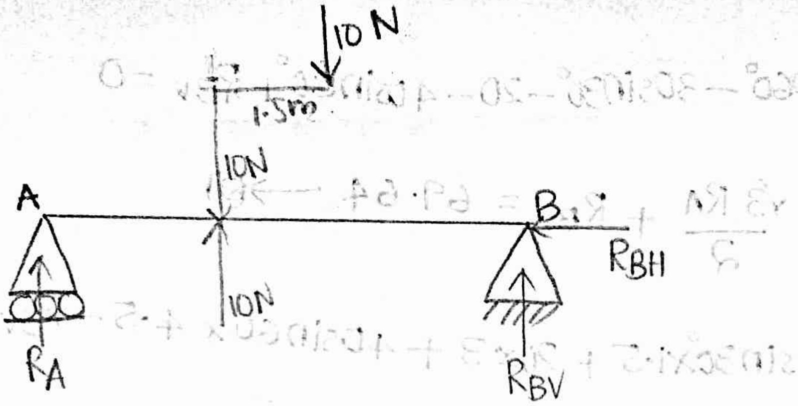
$$\theta = \tan^{-1}\left(\frac{R_{BV}}{R_{BH}}\right)$$

$$= \tan^{-1}\left(\frac{39.73}{23.25}\right)$$

$$= \underline{\underline{59.66^\circ}}$$

Q:2 Find the support reactions at A and B.





$\Sigma H=0 \Rightarrow R_{BH} = 0.$

$\Sigma V=0 \Rightarrow R_A - 10 + R_{BV} = 0$

$R_A + R_{BV} = 10 \longrightarrow (1)$

$\Sigma M=0 \Rightarrow 10 \times 3 + 10 \times 1.5 - R_{BV} \times 6 = 0$ 30+15

$\cancel{R_{BX} = \frac{30}{6} = 5N}$ $R_{BV} = \frac{45}{6} = 7.5N$

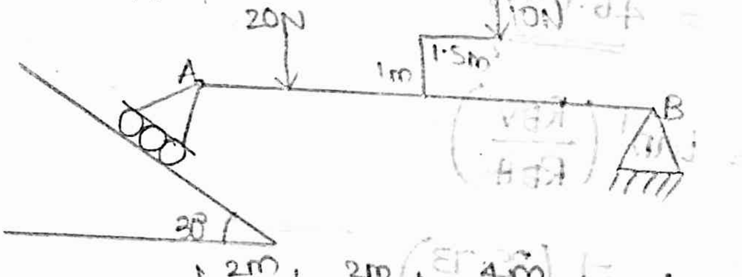
Sub. in (1),

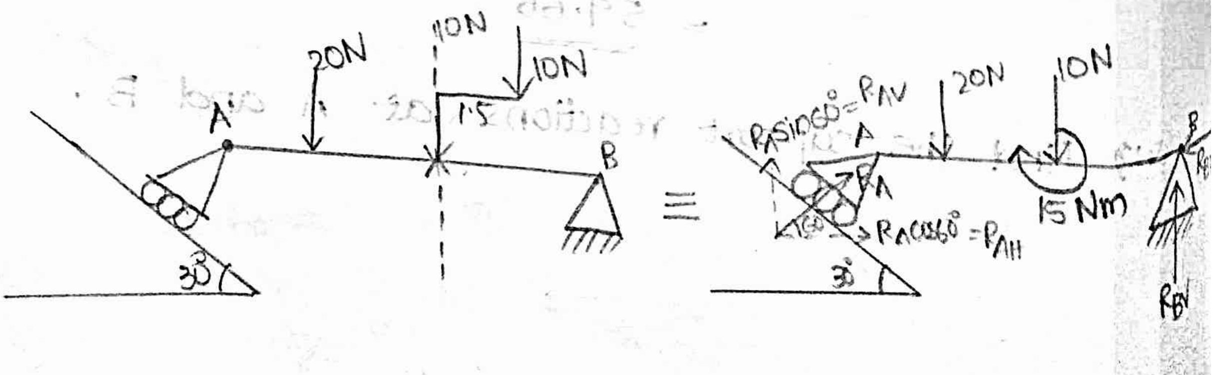
$R_A = 2.5N.$

$\cancel{R_B = 10N}$ $R_B = 7.5N$

19/9/2019.

Q: Find the support reactions at A and B.





$\Sigma H = 0$

$$R_{AH} - R_{BH} = 0 \longrightarrow (1)$$

$$\Sigma V = 0 : R_{AV} + R_{BV} - 20 - 10 = 0 \longrightarrow (2)$$

$$\Sigma M = 0 : 20 \times 2 + 10 \times 4 + 15 - R_{BV} \times 8 = 0$$

$$R_{BV} = 11.875\ N$$

$$R_{AV} = 30 - R_{BV}$$

$$= 18.125\ N$$

$$R_{AV} = R_A \sin 60^\circ$$

$$R_A = 20.929\ N$$

$$R_{AH} = R_A \cos 60^\circ$$

$$= 10.464\ N$$

$$\therefore R_{BH} = 10.464\ N$$

$$R_B = \sqrt{(R_{BH})^2 + (R_{BV})^2} = 15.827\ N$$

$$\theta = \tan^{-1}\left(\frac{R_{BV}}{R_{BH}}\right)$$

$$= \tan^{-1}\left(\frac{11.875}{10.464}\right)$$

$$= 48.61^\circ$$